## **Loseley Fields Primary School**

### Progression of Mathematical Calculations

This document has been written to set out the progression of skills in mathematical calculations taught at Loseley Fields. The main aim is to provide a clear picture for leaders, teachers, parents and governors in terms of the skills being taught in order to offer clear progression; both within year groups and across the school.

The document shows the progression in two directions: horizontally within a year group and vertically across year groups. Horizontal progress is shown through the transition from use of concrete apparatus to pictorial diagrams, before moving in to abstract representations of mathematical learning. Vertical progress is charted in terms of matching taught strategies and skills to the requirements of the National Curriculum for each individual year group.

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#### KEY STAGE 1

Children develop the core ideas that underpin all calculation. They begin by connecting calculation with counting on and counting back, but they should learn that understanding wholes and parts will enable them to calculate efficiently and accurately, and with greater flexibility. They learn how to use an understanding of 10s and 1s to develop their calculation strategies, especially in addition and subtraction.

**Key language:** whole, part, ones, ten, tens, number bond, add, addition, plus, total, altogether, subtract, subtraction, find the difference, take away, minus, less, more, group, share, equal, equals, is equal to, groups, equal groups, times, multiply, multiplied by, divide, share, shared equally, times-table

Addition and subtraction: Children first learn to Multiplication and division: Children develop an Fractions: In Year 1, children encounter halves and connect addition and subtraction with counting, but awareness of equal groups and link this with quarters, and link this with their understanding of they soon develop two very important skills: an counting in equal steps, starting with 2s, 5s and 10s. sharing. They experience key spatial representations of these fractions, and learn to recognise examples understanding of parts and wholes, and an In Year 2, they learn to connect the language of understanding of unitising 10s, to develop efficient equal groups with the mathematical symbols for and non-examples, based on their awareness of and effective calculation strategies based on known multiplication and division. equal parts of a whole. number bonds and an increasing awareness of place They learn how multiplication and division can be In Year 2, they develop an awareness of unit value. Addition and subtraction are taught in a way related to repeated addition and repeated fractions and experience non-unit fractions, and they that is interlinked to highlight the link between the subtraction to find the answer to the calculation. learn to write them and read them in the common two operations. In this key stage, it is vital that children explore and format of numerator and denominator. A key idea is that children will select methods and experience a variety of strong images and approaches based on their number sense. For manipulative representations of equal groups, example, in Year 1, when faced with 15 - 3 and including concrete experiences as well as abstract 15 – 13, they will adapt their ways of approaching the calculations. calculation appropriately. The teaching should Children begin to recall some key multiplication always emphasise the importance of mathematical facts, including doubles, and an understanding of thinking to ensure accuracy and flexibility of the 2, 5 and 10 times-tables and how they are related approach, and the importance of using known to counting. number facts to harness their recall of bonds within 20 to support both addition and subtraction methods. In Year 2, they will start to see calculations presented in a column format, although this is not expected to be formalised until KS2. We show the column method in Year 2 as an option; teachers may not wish to include it until Year 3.

		Year 1	
Year 1 Addition	Concrete	Pictorial	Abstract
Counting and adding more	Children add one more person or object to a group to find one more.	Children add one more cube or counter to a group to represent one more.	Use a number line to understand how to link counting on with finding one more.
		One more than 4 is 5.	One more 0 1 2 3 4 5 6 7 8 9 10 One more than 6 is 7. 7 is one more than 6.
			Learn to link counting on with adding more than one. 0  1  2  3  4  5  6  7  8  9  10 5 + 3 = 8
Understanding part-part-whole relationship	Sort people and objects into parts and understand the relationship with the whole.	Children draw to represent the parts and understand the relationship with the whole.	Use a part-whole model to represent the numbers. 10 6 4 6 + $4$ = $10$
	The parts are 2 and 4. The whole is 6.	The parts are 1 and 5. The whole is 6.	6 + 4 = 10

Knowing and finding number bonds within 10	Break apart a group and put back together to find and form number bonds. 3 + 4 = 7 6 = 2 + 4	Use five and ten frames to represent key number bonds. 5 = 4 + 1 $10 = 7 + 3$	Use a part-whole model alongside other representations to find number bonds. Make sure to include examples where one of the parts is zero. a) 4 - 0 4 4 4 4 4 4 4 4 4 4
Understanding teen numbers as a complete 10 and some more	Complete a group of 10 objects and count more.	Use a ten frame to support understanding of a complete 10 for teen numbers.	1 ten and 3 ones equal 13. 10 + 3 = 13

Adding by counting on	Children use knowledge of counting to 20 to find a total by counting on using people or objects. 8 on the bus 9 10 11	Children use counters to support and represent their counting on strategy.	Children use number lines or number tracks to support their counting on strategy. 7 7 + 5 =
Adding the 1s	Children use bead strings to recognise how to add the 1s to find the total efficiently. 2 + 3 = 5 12 + 3 = 15	Children represent calculations using ten frames to add a teen and 1s. 2 + 3 = 5 12 + 3 = 15	Children recognise that a teen is made from a 10 and some 1s and use their knowledge of addition within 10 to work efficiently. 3 + 5 = 8 So, 13 + 5 = 18
Understanding 10s and 1s	Group objects into 10s and 1s.	Understand 10s and 1s equipment, and link with visual representations on ten frames.	Represent numbers on a place value grid, using equipment or numerals. Tens Ones 3 2 Tens Ones 4 3

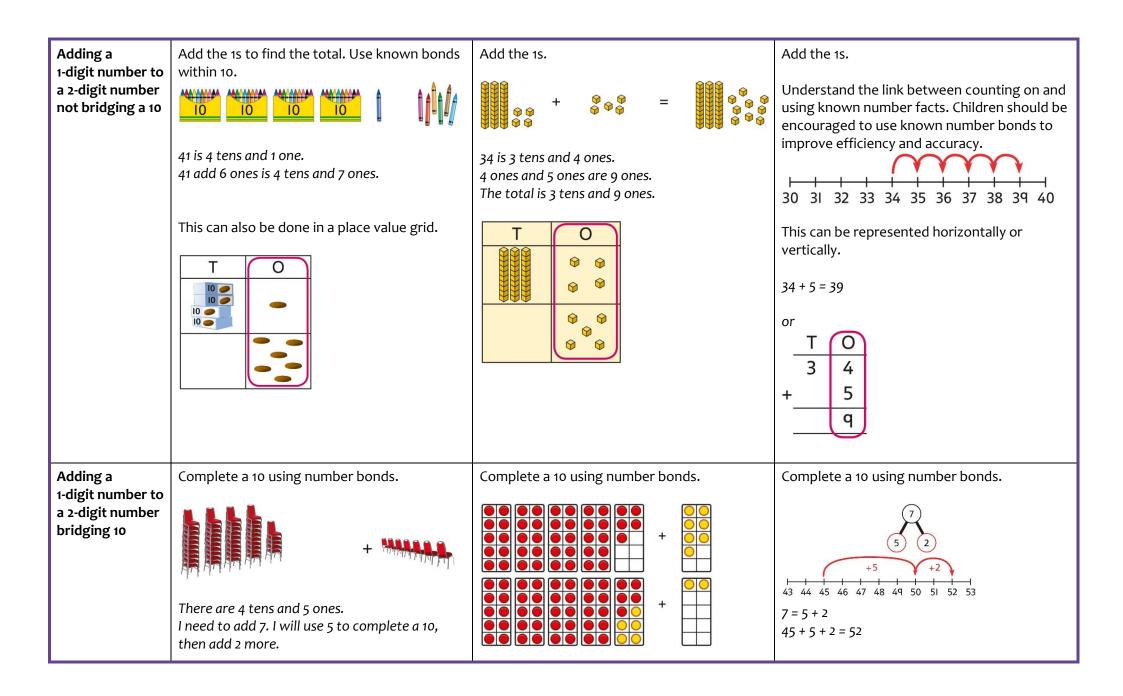
Year 1 Subtraction	Concrete	Pictorial	Abstract
Counting back and taking away	Children arrange objects and remove to find how many are left. 1 less than 6 is 5. 6 subtract 1 is 5.	Children draw and cross out or use counters to represent objects from a problem.	Children count back to take away and use a number line or number track to support the method. 876 $876$ $9-3=6$
Finding a missing part, given a whole and a part	Children separate a whole into parts and understand how one part can be found by subtraction. $\qquad \qquad \qquad$	Children represent a whole and a part and understand how to find the missing part by subtraction. 5 - 4 = 5	Children use a part-whole model to support the subtraction to find a missing part. 7 7 3 7-3=? Children develop an understanding of the relationship between addition and subtraction facts in a part-whole model. - - = + = + = + = + =

Finding the difference	Arrange two groups so that the difference between the groups can be worked out.	Represent objects using sketches or counters to support finding the difference.	Children understand 'find the difference' as subtraction.
	Image: Second state sta	5 - 4 = 1 The difference between 5 and 4 is 1.	$ \begin{array}{c}  & & & & \\  & & & & \\  & & & & \\  & & & &$
Subtraction within 20	Understand when and how to subtract 1s efficiently. Use a bead string to subtract 1s efficiently. 5-3=2 15-3=12	Understand when and how to subtract 1s efficiently. 5 - 3 = 2 15 - 3 = 12	Understand how to use knowledge of bonds within 10 to subtract efficiently. 5 - 3 = 2 15 - 3 = 12
Subtracting 10s and 1s	For example: 18 – 12 Subtract 12 by first subtracting the 10, then the remaining 2.	For example: 18 – 12 Use ten frames to represent the efficient method of subtracting 12.	Use a part-whole model to support the calculation. 14 10 19 - 14 19 - 10 = 9 9 - 4 = 5 So, $19 - 14 = 5$

Subtraction bridging 10 using number bonds	For example: 12 – 7 Arrange objects into a 10 and some 1s, then decide on how to split the 7 into parts.	Represent the use of bonds using ten frames.	Use a number line and a part-whole model to support the method. 13-5 $\begin{pmatrix} 5\\ 2\\ 3\\ -2\\ -2\\ -3\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 13$
Year 1 Multiplication	Concrete	Pictorial	Abstract
Describe equal groups using words	Children arrange objects in equal and unequal groups and understand how to recognise whether they are equal. A B C C C C C C C C C C C C C C C C C C	Children draw and represent equal and unequal groups.	Three equal groups of 4. Four equal groups of 3.
Finding the total of equal groups by counting in 2s, 5s and 10s	There are 5 pens in each pack 510152025303540	100 squares and ten frames support counting in 2s, 5s and 10s.	Use a number line to support repeated addition through counting in 2s, 5s and 1os. 10  10  10  10  10  10  10  10

Year 1 Division	Concrete	Pictorial	Abstract
Grouping	Learn to make equal groups from a whole and find how many equal groups of a certain size can be made.	Represent a whole and work out how many equal groups.	Children may relate this to counting back in steps of 2, 5 or 10.
	Sort a whole set people and objects into equal groups.	00000 00000	
		There are 10 in total. There are 5 in each group. There are 2 groups.	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
	There are 10 children altogether. There are 2 in each group. There are 5 groups.		
Sharing	Share a set of objects into equal parts and work out how many are in each part.	Sketch or draw to represent sharing into equal parts. This may be related to fractions.	10 shared into 2 equal groups gives 5 in each group.

	Year 2			
Year 2 Addition	Concrete	Pictorial	Abstract	
Bridging	Bridging the 10 using number bonds Children use a bead string to complete a 10 and understand how this relates to the addition. 7 add 3 makes 10. So, 7 add 5 is 10 and 2 more.	Bridging the 10 using number bonds Children use counters to complete a ten frame and understand how they can add using knowledge of number bonds to 10.	Bridging the 10 using number bonds Use a part-whole model and a number line to support the calculation. 4 1 3 9 + 4 = 13	
Adding 10s	Use known bonds and unitising to add 10s. ())) ()) ()) ()) ()) ()) ()) ()) ()) ()	Use known bonds and unitising to add 10s. Use known bonds and unitising to add 10s.	Use known bonds and unitising to add 10s. $ \begin{array}{r} 7\\ 4\\ 3\\ 4+3=1\\ 4+3=7\\ 4 \text{ tens} + 3 \text{ tens} = 7 \text{ tens}\\ 40+30=70 \end{array} $	



Adding a 1-digit number to	Exchange 10 ones for 1 ten.	Exchange 10 ones for 1 ten.	Exchange 10 ones for 1 ten.
a 2-digit number using exchange			$ \begin{array}{c c} T & O \\ \hline 2 & 4 \\ + & 8 \\ \hline 2 \\ \hline 1 \\ \end{array} $
			T     O       2     4       8       3     2       1
Adding a	Add the 10s and then recombine.	Add the 10s and then recombine.	Add the 10s and then recombine.
multiple of 10 to a 2-digit number		$ \begin{array}{c} & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & $	37 + 20 = ? 30 + 20 = 50 50 + 7 = 57
	27 is 2 tens and 7 ones. 50 is 5 tens.		37 + 20 = 57
	There are 7 tens in total and 7 ones. So, 27 + 50 is 7 tens and 7 ones.	66 is 6 tens and 6 ones. 66 + 10 = 76	
		A too square can support this understanding. $\begin{array}{c ccccccccccccccccccccccccccccccccccc$	

Adding a multiple of 10 to a 2-digit number using columns	Add the 10s using a place value grid to support. TOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOO	Add the 10s using a place value grid to support. TO O O O O O O O O O O O O O	Add the 10s represented vertically. Children must understand how the method relates to unitising of 10s and place value. $\begin{array}{r} \hline \\ \hline $
Adding two 2-digit numbers	Add the 10s and 1s separately. Add the 10s and 1s separately. 3 + 3 = 8 There are 8 ones in total. 3 + 2 = 5 There are 5 tens in total. 35 + 23 = 58	Add the 10s and 1s separately. Use a part-whole model to support. 32 + 11 $11 = 10 + 1$ $32 + 10 = 42$ $42 + 1 = 43$ $32 + 11 = 43$	Add the 10s and the 1s separately, bridging 10s where required. A number line can support the calculations. $\frac{+10}{17} + \frac{10}{17} + \frac{1}{25} + \frac{1}{25}$

Adding two 2-digit numbers using a place value grid	Add the 1s. Then add the 1os.   Tens Ones   Image: transmitted state   Image: transmitted state <	Add the 1s. Then add the 1os. T = 0 3 = 2 + 1 = 4 6 T = 0 3 = 2 + 1 = 4 6 + 1 = 4 6
Adding two 2-digit numbers with exchange	Add the 1s. Exchange 10 ones for a ten. Then add the 1os.         Tens       Ones         9       6         9       6         9       6         9       7         9       7         10       10         10 </th <th>Add the 1s. Exchange 10 ones for a ten. Then add the 1os. <math display="block">\begin{array}{r} T \\ \hline 0 \\ \hline 3 \\ \hline + 2 \\ \hline - \\ \hline - \\ \hline \\ \hline 0 \\ \hline 3 \\ \hline 6 \\ \hline 2 \\ \hline - \\ \hline \end{array}</math></th>	Add the 1s. Exchange 10 ones for a ten. Then add the 1os. $\begin{array}{r} T \\ \hline 0 \\ \hline 3 \\ \hline + 2 \\ \hline - \\ \hline - \\ \hline \\ \hline 0 \\ \hline 3 \\ \hline 6 \\ \hline 2 \\ \hline - \\ \hline \end{array}$

Year 2 Subtraction	Concrete	Pictorial	Abstract
Subtracting multiples of 10	Use known number bonds and unitising to subtract multiples of 10.	Use known number bonds and unitising to subtract multiples of 10.	Use known number bonds and unitising to subtract multiples of 10.
	Q Q X X X X X X X	100           30	7 70 2 5 20 50
	8 subtract 6 is 2. So, 8 tens subtract 6 tens is 2 tens.	10 – 3 = 7 So, 10 tens subtract 3 tens is 7 tens.	7 tens subtract 5 tens is 2 tens. 70 – 50 = 20
Subtracting a single-digit number	Subtract the 1s. This may be done in or out of a place value grid. $\begin{array}{c} \hline 0 \\ \hline 0$	Subtract the 1s. This may be done in or out of a place value grid.	Subtract the 1s. Understand the link between counting back and subtracting the 1s using known bonds. $\begin{array}{r} & & \\ \hline & & \\ 30 & 31 & 32 & 33 & 34 & 35 & 36 & 37 & 38 & 39 & 40 \\ \hline & & & \\ \hline \end{array} $
Subtracting a single-digit number bridging 10	Bridge 10 by using known bonds.	Bridge 10 by using known bonds.	Bridge 10 by using known bonds.
	35 - 6 I took away 5 counters, then 1 more.	35 – 6 First, I will subtract 5, then 1.	24 - 6 = ? 24 - 4 - 2 = ?

Subtracting a single-digit number using exchange	Exchange 1 ten for 10 ones. This may be done in or out of a place value grid.	Exchange 1 ten for 10 ones.	Exchange 1 ten for 10 ones. $T \bigcirc 12 15 \\ - 7 \\ - 7 \\ - 8 \\ - 7 \\ - 7 \\ - 8 \\ - 7 \\ - 7 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 8 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 1 \\ - 8 \\ - 7 \\ - 7 \\ - 8 \\ - 7 \\ - 7 \\ - 8 \\ - 7 \\ - 7 \\ - 8 \\ - 7 \\ - 7 \\ - 8 \\ - 7 \\ - 8 \\ - 7 \\ - 8 \\ - 8 \\ - 7 \\ - 8 \\$
Subtracting a 2-digit number	Subtract by taking away.	Subtract the 10s and the 1s. This can be represented on a 100 square. 1       2       3       4       5       6       7       8       9       10         11       12       13       14       15       16       17       18       19       20         21       22       23       24       25       26       27       28       29       30         31       32       33       34       35       36       37       38       39       40         41       42       43       44       45       46       47       148       49       50         51       52       53       54       55       56       57       58       59       60         61       62       63       64       65       66       67       68       69       70         71       72       73       74       75       76       77       78       79       80         81       82       83       84       85       86       87       88       89       100         91       92       93       94       95       96       97       98       99       <	Subtract the 10s and the 1s. This can be represented on a number line. -10 $-10$

Subtracting a 2-digit number using place value and columns	Subtract the 1s. Then subtract the 1os. This may be done in or out of a place value grid. T O $O$ $O$ $O$ $O$ $O$ $O$ $O$ $O$ $O$	Subtract the 1s. Then subtract the 1os.	Using column subtraction, subtract the 1s. Then subtract the 1os. $T \bigcirc 4$ 5 -12 3 $T \bigcirc 6$ 4 5 -12 3 3 3
Subtracting a 2-digit number with exchange		Exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 1os. Tens Ones Tens Ones Tens Ones Tens Ones Tens Ones Tens Ones	Using column subtraction, exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 1os. $\frac{T O}{4 5}$ $-2 7$ $\frac{T O}{3 \# 5}$ $-2 7$ $\frac{T O}{3 \# 5}$ $-2 7$ $\frac{T O}{3 \# 5}$ $-2 7$ $\frac{8}{8}$ $\frac{T O}{3 \# 5}$ $-2 7$ $\frac{8}{18}$

Year 2 Multiplication	Concrete	Pictorial	Abstract
Equal groups and repeated addition	Recognise equal groups and write as repeated addition and as multiplication.	Recognise equal groups using standard objects such as counters and write as repeated addition and multiplication.	Use a number line and write as repeated addition and as multiplication.
	3 groups of 5 chairs 15 chairs altogether	3 groups of 5 15 in total	5 + 5 + 5 = 15 3 × 5 = 15
Using arrays to represent multiplication	Understand the relationship between arrays, multiplication and repeated addition.	Understand the relationship between arrays, multiplication and repeated addition.	Understand the relationship between arrays, multiplication and repeated addition.
and support understanding			
	4 groups of 5	4 groups of 5 5 groups of 5	5 × 5 = 25
Understanding commutativity	Use arrays to visualise commutativity.	Form arrays using counters to visualise commutativity. Rotate the array to show that	Use arrays to visualise commutativity.
		orientation does not change the multiplication.	
	I can see 6 groups of 3. I can see 3 groups of 6.	This is 2 groups of 6 and also 6 groups of 2.	4 + 4 + 4 + 4 + 4 = 20 5 + 5 + 5 + 5 = 20 4 × 5 = 20 and 5 × 4 = 20

Learning ×2, ×5 and ×10 table facts	Develop an understanding of how to unitise groups of 2, 5 and 10 and learn corresponding times-table facts.	Understand how to relate counting in unitised groups and repeated addition with knowing key times-table facts.	Understand how the times-tables increase and contain patterns.
		00000000	
		000000000	10 10 10
	3 groups of 10 10, 20, 30 3 × 10 = 30	10 + 10 + 10 = 30	
	3 × 10 - 30	3 × 10 = 30	
			10 10 10 10 10 10 10 10 10 10 10 10 10 1
			5 × 10 = 50 6 × 10 = 60

Year 2 Division	Concrete	Pictorial	Abstract
Sharing equally	Start with a whole and share into equal parts, one at a time.	Represent the objects shared into equal parts using a bar model.	Use a bar model to support understanding of the division.
	<ul> <li>12 shared equally between 2.</li> <li>12 shared equally between 2.</li> <li>They get 6 each.</li> <li>Start to understand how this also relates to grouping. To share equally between 3 people, take a group of 3 and give 1 to each person.</li> <li>Keep going until all the objects have been</li> </ul>	20 shared into 5 equal parts. There are 4 in each part.	18 ÷ 2 = 9
	shared shared shared		
	They get 5 ach. 15 shared equally between 3. They get 5 each.		

Grouping equally	Understand how to make equal groups from a whole.	Understand the relationship between grouping and the division statements.	Understand how to relate division by grouping to repeated subtraction.
	?	$12 \div 3 = 4$	
	8 divided into 4 equal groups. There are 2 in each group.	$12 \div 4 = 3$	0 1 2 3 4 5 6 7 8 9 10 11 12
		$12 \div 6 = 2$	There are 4 groups now.
		$12 \div 2 = 6$	12 divided into groups of 3. 12 ÷ 3 = 4
			There are 4 groups.
Using known times-tables to solve divisions	Understand the relationship between multiplication facts and division.	Link equal grouping with repeated subtraction and known times-table facts to support division.	Relate times-table knowledge directly to division.
		40 divided by 4 is 10.	$I \times I0 = I0$ $2 \times I0 = 20$ $3 \times I0 = 30$ $4 \times I0 = 40$ $5 \times I0 = 50$ $6 \times I0 = 60$ $7 \times I0 = 70$ $8 \times I0 = 80$ $I \text{ used the I0}$ $times-table$ $to help me.$ $3 \times I0 = 30.$
	4 groups of 5 cars is 20 cars in total. 20 divided by 4 is 5.	Use a bar model to support understanding of the link between times-table knowledge and division.	I know that 3 groups of 10 makes 30, so I know that 30 divided by 10 is 3.
		60 10 10	3 × 10 = 30 so 30 ÷ 10 = 3

#### LOWER KEY STAGE 2

In Years 3 and 4, children develop the basis of written methods by building their skills alongside a deep understanding of place value. They should use known addition/subtraction and multiplication/division facts to calculate efficiently and accurately, rather than relying on counting. Children use place value equipment to support their understanding, but not as a substitute for thinking.

Key language: partition, place value, tens, hundreds, thousands, column method, whole, part, equal groups, sharing, grouping, bar model

Addition and subtraction: In Year 3 especially, the column methods are built up gradually. Children will develop their understanding of how each stage of the calculation, including any exchanges, relates to place value. The example calculations chosen to introduce the stages of each method may often be more suited to a mental method. However, the examples and the progression of the steps have been chosen to help children develop their fluency in the process, alongside a deep understanding of the concepts and the numbers involved, so that they can apply these skills accurately and efficiently to later calculations. The class should be encouraged to compare mental and written methods for specific calculations, and children should be encouraged at every stage to make choices about which methods to apply. In Year 4, the steps are shown without such fine

detail, although children should continue to build their understanding with a secure basis in place value. In subtraction, children will need to develop their understanding of exchange as they may need to exchange across one or two columns. By the end of Year 4, children should have developed fluency in column methods alongside a deep understanding, which will allow them to progress confidently in upper Key Stage 2.

Multiplication and division: Children build a solid grounding in times-tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35. Children develop key skills to support multiplication methods: unitising, commutativity, and how to use partitioning effectively. Unitising allows children to use known facts to multiply and divide multiples of 10 and 100 efficiently. Commutativity gives children flexibility in applying known facts to calculations and problem solving. An understanding of partitioning allows children to extend their skills to multiplying and dividing 2- and 3-digit numbers by a single digit. Children develop column methods to support multiplications in these cases.

For successful division, children will need to make choices about how to partition. For example, to divide 423 by 3, it is effective to partition 423 into 300, 120 and 3, as these can be divided by 3 using known facts.

Children will also need to understand the concept of remainder, in terms of a given calculation and in terms of the context of the problem. **Fractions:** Children develop the key concept of equivalent fractions, and link this with multiplying and dividing the numerators and denominators, as well as exploring the visual concept through fractions of shapes. Children learn how to find a fraction of an amount, and develop this with the aid of a bar model and other representations alongside. in Year 3, children develop an understanding of how to add and subtract fractions with the same denominator and find complements to the whole. This is developed alongside an understanding of fractions as numbers, including fractions greater than 1. In Year 4, children begin to work with fractions greater than 1.

Decimals are introduced, as tenths in Year 3 and then as hundredths in Year 4. Children develop an understanding of decimals in terms of the relationship with fractions, with dividing by 10 and 100, and also with place value.

		Year 3	
Year 3 Addition	Concrete	Pictorial	Abstract
Understanding 100s	Understand the cardinality of 100, and the link with 10 tens. Use cubes to place into groups of 10 tens.	Unitise 100 and count in steps of 100.	Represent steps of 100 on a number line and a number track and count up to 1,000 and back to 0.
Understanding place value to 1,000	Unitise 100s, 10s and 1s to build 3-digit numbers.	Use equipment to represent numbers to 1,000.	Represent the parts of numbers to 1,000 using a part-whole model. 215 200 $10$ $5215 = 200 + 10 + 5Recognise numbers to 1,000 represented on anumber line, including those betweenintervals.$

Adding 100s	Use known facts and unitising to add multiples of 100. 100 bricks + 100 bricks bricks + 100 bricks 3 + 2 = 5 3 hundreds + 2 hundreds = 5 hundreds 300 + 200 = 500	Use known facts and unitising to add multiples of 100.	Use known facts and unitising to add multiples of 100. Represent the addition on a number line. Use a part-whole model to support unitising. 3 + 2 = 5 300 + 200 = 500
3-digit number + 1s, no exchange or bridging	Use number bonds to add the 1s. Use number bonds to add the 1s. 1 + 4 = ? Now there are 4 + 4 ones in total. 4 + 4 = 8 214 + 4 = 218	Use number bonds to add the 1s. $ \begin{array}{c c} H & T & O \\ \hline                                  $	Understand the link with counting on. 245 + 4 245 + 4 245 + 4 245 + 46 + 247 + 248 + 249 + 250 Use number bonds to add the 1s and understand that this is more efficient and less prone to error. 245 + 4 = ? I will add the 1s. 5 + 4 = 9 So, $245 + 4 = 249$

3-digit number + 1s with exchange	Understand that when the 1s sum to 10 or more, this requires an exchange of 10 ones for 1 ten.	Exchange 10 ones for 1 ten where needed. Use a place value grid to support the understanding.	Understand how to bridge by partitioning to the 1s to make the next 10.
	Children should explore this using unitised objects or physical apparatus.	HTOHTOHTOHTOSSSSSSSSHTOSSSSSSSSHTOSSSSSSSSHTOSSSSSSSSHTOSSHTOSS <t< td=""><td>7 = 7 <math display="block">5 = 7</math> <math display="block">135 + 7 = ?</math> <math display="block">135 + 7 = ?</math> <math display="block">135 + 5 + 2 = 142</math> Ensure that children understand how to add 1s bridging a 100. <math display="block">198 + 5 = ?</math> <math display="block">198 + 2 + 3 = 203</math></td></t<>	7 = 7 $5 = 7$ $135 + 7 = ?$ $135 + 7 = ?$ $135 + 5 + 2 = 142$ Ensure that children understand how to add 1s bridging a 100. $198 + 5 = ?$ $198 + 2 + 3 = 203$
		H T O 135 + 7 = 142	

3-digit number + 10s, no exchange	Calculate mentally by forming the number bond for the 10s.	Calculate mentally by forming the number bond for the 10s.	Calculate mentally by forming the number bond for the 10s.
		351 + 30 = ?	753 + 40 I know that 5 + 4 = 9 So, 50 + 40 = 90 753 + 40 = 793
	234 + 50 There are 3 tens and 5 tens altogether. 3 + 5 = 8 In total there are 8 tens. 234 + 50 = 284	5 tens + 3 tens = 8 tens 351 + 30 = 381	
3-digit number + 10s, with exchange	Understand the exchange of 10 tens for 1 hundred.	Add by exchanging 10 tens for 1 hundred. 184 + 20 = ? H T O 1000000000000000000000000000000000000	Understand how the addition relates to counting on in 10s across 100. $\begin{array}{c} & & \\ \hline \\ 184 \\ 190 \\ 184 \\ 200 \\ 184 \\ 200 \\ 184 \\ 200 \\ 184 \\ 200 \\ 184 \\ 200 \\ 184 \\ 200 \\ 200$
		184 + 20 = 204	385 + 50 There are 8 tens and 5 tens. That is 13 tens. 385 + 50 = 300 + 130 + 5 385 + 50 = 435

3-digit number + 2-digit number	Use place value equipment to make and combine groups to model addition.	Use a place value grid to organise thinking and adding of 1s, then 10s.	Use the vertical column method to represent the addition. Children must understand how this relates to place value at each stage of the calculation.
3-digit number + 2-digit number, exchange required	Use place value equipment to model addition and understand where exchange is required. Use place value counters to represent 154 + 72. Use this to decide if any exchange is required. There are 5 tens and 7 tens. That is 12 tens so 1 will exchange.	Represent the required exchange on a place value grid using equipment. 275 + 16 = ? H T T T T T T T T T T T T T T T T T T	Use a column method with exchange. Children must understand how the method relates to place value at each stage of the calculation. $\frac{H T O}{2 7 5} + \frac{1 6}{1 6} + \frac{1 6}{9 1} + \frac{1 6}{1 6} + \frac{1 6}{9 1} + \frac{1 6}{1 6} + \frac{1 6}{9 1} + \frac{1 6}{1 6} + \frac{1 6}{2 9 1} + \frac{1 6}{1 6} + \frac{1 6}{2 9 1} + \frac{1 6}{1 6} + \frac{1 6}{2 9 1} + \frac{1 6}{1 6} + \frac{1 6}{2 9 1} + \frac{1 6}{1 6} + \frac{1 6}{2 9 1} + \frac{1 6}{1 6} + \frac{1 6}{2 9 1} + \frac{1 6}{1 6} + \frac{1 6}{2 9 1} + \frac{1 6}{1 6} + \frac{1 6}{2 9 1} + \frac{1 6}{1 6} + \frac{1 6}{2 9 1} + \frac{1 6}{1 6} + \frac{1 6}{2 9 1} + \frac{1 6}{1 6} + \frac{1 6}{2 9 1} + \frac{1 6}{1 6} + \frac{1 6}{2 9 1} + \frac{1 6}{1 6} + \frac{1 6}{2 9 1} + \frac{1 6}{1 6} + $
		how the method relates to place value. Children should be encouraged at every stage to select methods that are accurate and efficient.	

3-digit number + 3-digit number, no exchange	Use place value equipment to make a representation of a calculation. This may or may not be structured in a place value grid. 326 + 541 is represented as: 1000 + 10000 + 1000 + 1000 + 10000 + 1000 +	Represent the place value grid with equipment to model the stages of column addition.	Use a column method to solve efficiently, using known bonds. Children must understand how this relates to place value at every stage of the calculation.
3-digit number + 3-digit number, exchange required	Use place value equipment to enact the exchange required.	Model the stages of column addition using place value equipment on a place value grid.	Use column addition, ensuring understanding of place value at every stage of the calculation. $\frac{H T 0}{1 2 6} + \frac{2 1 7}{3 4 3}$ $\frac{H T 0}{1 2 6} + \frac{2 1 7}{4 3}$ $\frac{H T 0}{1 2 6} + \frac{2 1 7}{3 4 3}$ $\frac{126 + 217 = 343}{1 3 4 3}$ Note: Children should also study examples where exchange is required in more than one column, for example 185 + 318 = ?

Representing addition problems, and selecting appropriate methods	Encourage children to use their own drawings and choices of place value equipment to represent problems with one or more steps. These representations will help them to select appropriate methods.	Children understand and create bar models to represent addition problems. 275 + 99 = ? 374 275 qq 275 + 99 = 374	Use representations to support choices of appropriate methods. 275 qq I will add 100, then subtract 1 to find the solution. 128 + 105 + 83 = ? I need to add three numbers. 128 + 105 = 233 233 128 105 83 316 233 83
Year 3 Subtraction	Concrete	Pictorial	Abstract
Subtracting 100s	Use known facts and unitising to subtract multiples of 100.	Use known facts and unitising to subtract multiples of 100. 4 - 2 = 2 $400 - 200 = 200$	Understand the link with counting back in 1005. 100 200 300 400 500 400 - 200 = 200 Use known facts and unitising as efficient and accurate methods. 1 know that 7 - 4 = 3. Therefore, 1 know that 700 - 400 = 300.

3-digit number – 1s, no exchange	Use number bonds to subtract the 1s. 214 - 3 = ? 4 - 3 = 1 214 - 3 = 211	Use number bonds to subtract the 1s. $ \begin{array}{c c} H & T & O \\ \hline  & & & & \\ \hline  & & & \\$	Understand the link with counting back using a number line. Use known number bonds to calculate mentally. 476 - 4 = ? 476 400 70 6 6 - 4 = 2 476 - 4 = 472
3-digit number – 1s, exchange or bridging required	Understand why an exchange is necessary by exploring why 1 ten must be exchanged. Use place value equipment.	Represent the required exchange on a place value grid. 151 - 6 = ? H T O H T O H T O H T O N N N N N N N N N N N N N N N N N N N	Calculate mentally by using known bonds. 151 – 6 = ? 151 – 1 – 5 = 145

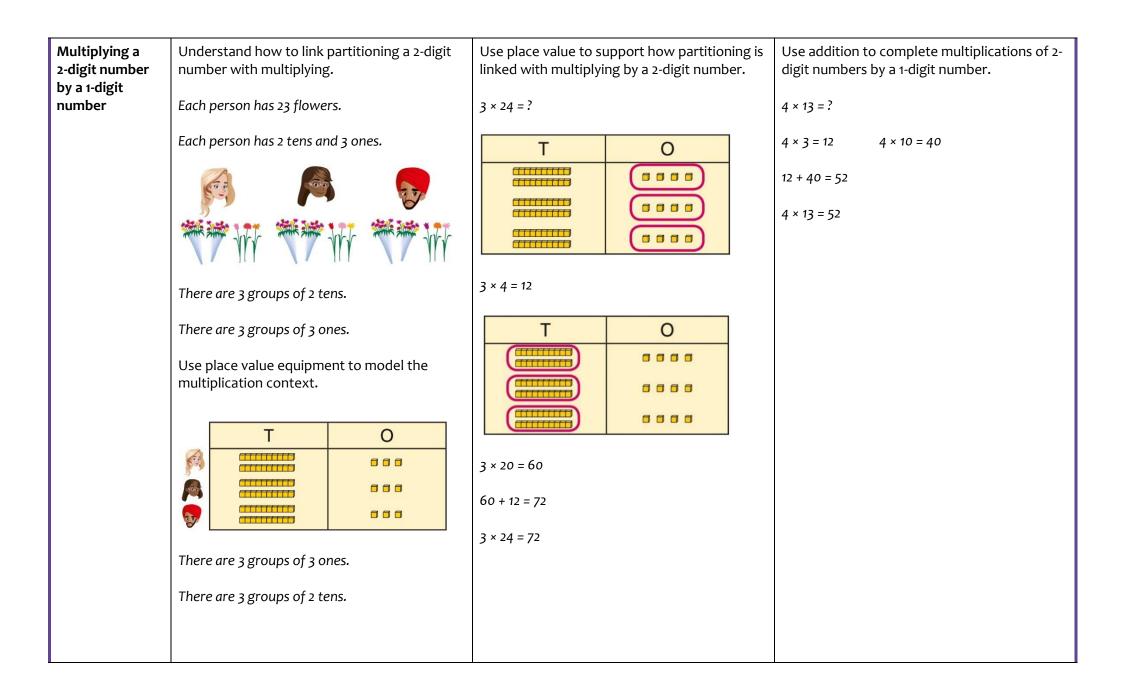
3-digit number – 10s, no exchange	Subtract the 10s using known bonds.	Subtract the 10s using known bonds. $\begin{array}{c c} H & T & O \\ \hline 0 & \hline $	Use known bonds to subtract the 10s mentally. 372 - 50 = ? 70 - 50 = 20 So, 372 - 50 = 322
3-digit number – 10s, exchange or bridging required	Use equipment to understand the exchange of 1 hundred for 10 tens.	Represent the exchange on a place value grid using equipment. 210 - 20 = ? $\frac{H T 0}{100}$ I need to exchange 1 hundred for 10 tens, to help subtract 2 tens. $\frac{H T 0}{100}$ 210 - 20 = 190	Understand the link with counting back on a number line. Use flexible partitioning to support the calculation. 235 - 60 = ? 235 - 60 = ? 235 = 100 + 130 + 5 235 = 100 + 130 + 5 235 - 60 = 100 + 70 + 5 = 175

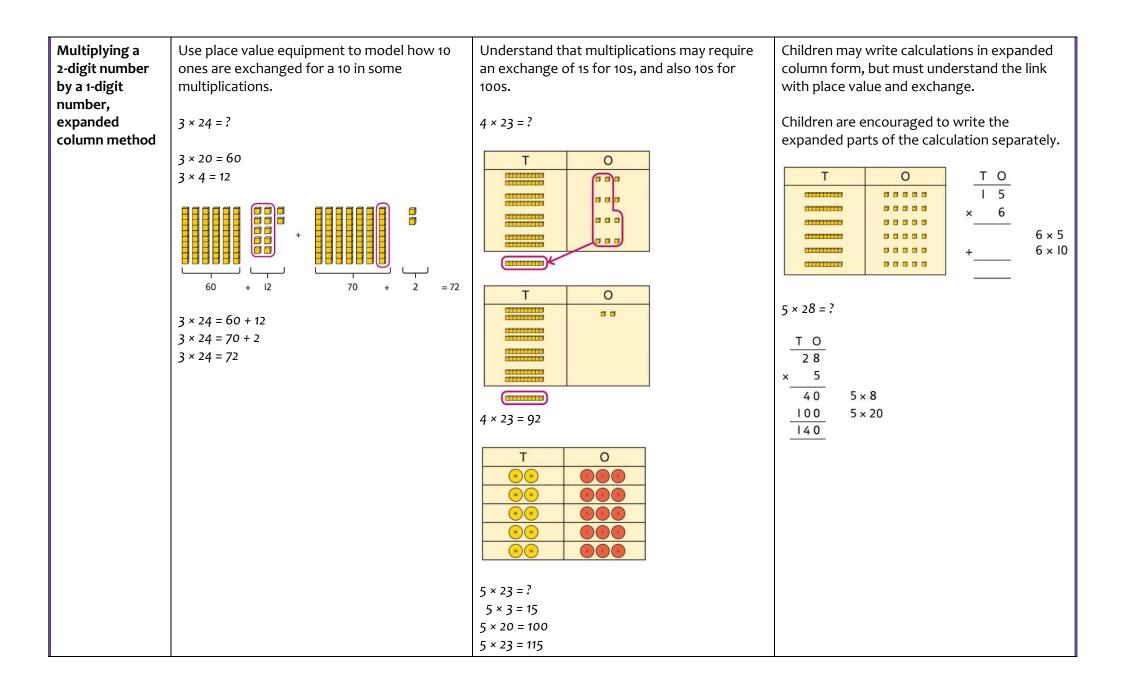
3-digit number – up to 3-digit number	Use place value equipment to explore the effect of splitting a whole into two parts, and understand the link with taking away.	Represent the calculation on a place value grid.	Use column subtraction to calculate accurately and efficiently. $\frac{H T O}{q q q}$ $-352$ $-7$ $\frac{H T O}{q q q}$ $-352$ $-47$ $\frac{H T O}{q q q}$ $-352$ $-47$ $\frac{H T O}{q q q}$ $-352$ $-647$
3-digit number – up to 3-digit number, exchange required	Use equipment to enact the exchange of 1 hundred for 10 tens, and 1 ten for 10 ones. $\longrightarrow \qquad \qquad$	Model the required exchange on a place value grid. 175 - 38 = ? I need to subtract 8 ones, so I will exchange a ten for 10 ones. H T O H T O	Use column subtraction to work accurately and efficiently. $\frac{H T 0}{1 6 \frac{1}{5} 5} - \frac{3 8}{1 3 7}$ $\frac{1}{175 - 38 = 137}$ If the subtraction is a 3-digit number subtract a 2-digit number, children should understand how the recording relates to the place value, and so how to line up the digits correctly. Children should also understand how to exchange in calculations where there is a zero in the 10s column.

Representing subtraction problems		Use bar models to represent subtractions. 'Find the difference' is represented as two bars for comparison. Team A 454 Team B 128 ? Bar models can also be used to show that a part must be taken away from the whole.	Children use alternative representations to check calculations and choose efficient methods. Children use inverse operations to check additions and subtractions. The part-whole model supports understanding. I have completed this subtraction. 525 - 270 = 255 I will check using addition. 525 - 270 = 255 I will check using addition. 525 - 270 = 255 I will check using addition.
Year 3 Multiplication	Concrete	Pictorial	Abstract
Understanding equal grouping and repeated addition	Children continue to build understanding of equal groups and the relationship with repeated addition. They recognise both examples and non- examples using objects.	Children recognise that arrays demonstrate commutativity.	Children understand the link between repeated addition and multiplication. $ \begin{array}{r} +3 \\ 0 \\ 3 \\ 0 \\ 3 \\ 6 \\ 9 \\ 12 \\ 15 \\ 18 \\ 21 \\ 24 \\ 12 \\ 15 \\ 18 \\ 21 \\ 24 \\ 12 \\ 15 \\ 18 \\ 21 \\ 24 \\ 24 \\ 24 \\ 24 \\ 24 \\ 24 \\ 24$

	本部部部部部部部部部部部部部部部部部部部部部部部部部部部部部部部部部部部部		$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
Using commutativity to support understanding of the times- tables	Understand how to use times-tables facts flexibly.	Understand how times-table facts relate to commutativity.	Understand how times-table facts relate to commutativity. I need to work out 4 groups of 7. I know that 7 × 4 = 28 so, I know that
	There are 6 groups of 4 pens. There are 4 groups of 6 bread rolls. I can use 6 × 4 = 24 to work out both totals.	6 × 4 = 24 4 × 6 = 24	4 groups of 7 = 28 and 7 groups of 4 = 28.

Understanding and using ×3, ×2, ×4 and ×8 tables.	Children learn the times-tables as 'groups of', but apply their knowledge of commutativity.	Children understand how the ×2, ×4 and ×8 tables are related through repeated doubling.	Children understand the relationship between related multiplication and division facts in known times-tables. $2 \times 5 = 10$ $5 \times 2 = 10$ $10 \div 5 = 2$ $10 \div 2 = 5$
Using known facts to multiply 10s, for example 3 × 40	Explore the relationship between known times-tables and multiples of 10 using place value equipment. <i>Make 4 groups of 3 ones.</i> <i>Make 4 groups of 3 tens.</i> <i>Make 4 groups of 3 tens.</i> <i>What is the same?</i> <i>What is different?</i>	Understand how unitising 10s supports multiplying by multiples of 10. Understand how unitising 10s supports multiplying by multiples of 10. Unitiplying by multiples of 10. Unit for the set of 10 to 10	Understand how to use known times-tables to multiply multiples of 10. $\begin{array}{r} +2 & +2 & +2 & +2 \\ \hline 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline +20 & +20 & +20 & +20 \\ \hline 0 & 10 & 20 & 30 & 40 & 50 & 60 & 70 & 80 \\ \hline 4 \times 2 = 8 \\ 4 \times 20 = 80 \end{array}$





Year 3 Division	Concrete	Pictorial	Abstract
Using times- tables knowledge to divide	Use knowledge of known times-tables to calculate divisions. 24 divided into groups of 8. There are 3 groups of 8.	Use knowledge of known times-tables to calculate divisions. <b>4</b> <b>4</b> <b>4</b> <b>5</b> <b>4</b> <b>5</b> <b>4</b> <b>5</b> <b>4</b> <b>5</b> <b>5</b> <b>6</b> <b>6</b> <b>6</b> <b>6</b> <b>6</b> <b>6</b> <b>6</b> <b>6</b>	Use knowledge of known times-tables to calculate divisions. I need to work out 30 shared between 5. I know that $6 \times 5 = 30$ so I know that $30 \div 5 = 6$ . A bar model may represent the relationship between sharing and grouping. 24 4 4 4 4 4 4 4 4 4

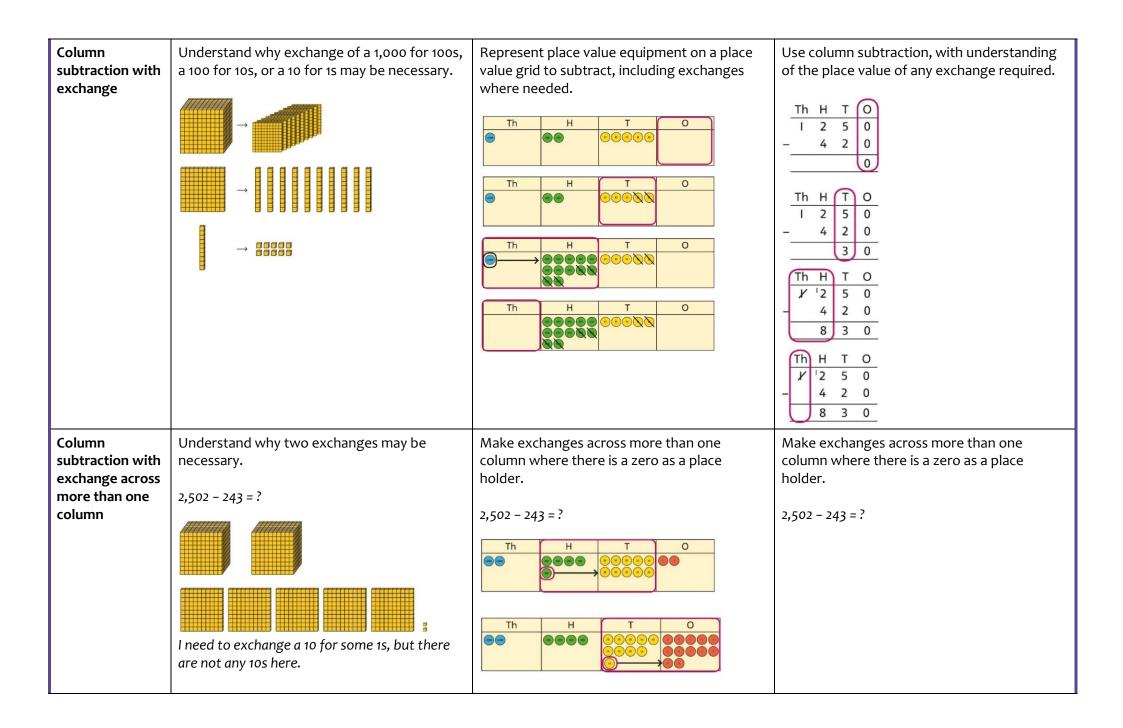
Understanding remainders	Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further.	Use images to explain remainders.	Understand that the remainder is what cannot be shared equally from a set. $22 \div 5 = ?$ $3 \times 5 = 15$ $4 \times 5 = 20$ $5 \times 5 = 25 \dots$ this is larger than 22 So, 22 ÷ 5 = 4 remainder 2
Using known facts to divide multiples of 10	Use place value equipment to understand how to divide by unitising. <i>Make</i> 6 ones divided by 3. Now make 6 tens divided by 3. What is the same? What is different?	Divide multiples of 10 by unitising.	Divide multiples of 10 by a single digit using known times-tables. 180 ÷ 3 = ? 180 is 18 tens. 18 divided by 3 is 6. 18 tens divided by 3 is 6 tens. 18 ÷ 3 = 6 180 ÷ 3 = 60
2-digit number divided by 1-digit number, no remainders	Children explore dividing 2-digit numbers by using place value equipment.	Children explore which partitions support particular divisions.	Children partition a number into 10s and 1s to divide where appropriate. $68$ $60 \div 2 = 30$ $8 \div 2 = 4$ $30 \div 4 = 34$ $68 \div 2 = 34$ Children partition flexibly to divide where appropriate.

	Then divide the 1s.	$42 = 30 + 12$ $42 \div 3 = 14$	$42 \div 3 = ?$ 42 = 40 + 2 I need to partition 42 differently to divide by 3. 42 = 30 + 12 $30 \div 3 = 10$ $12 \div 3 = 4$ 10 + 4 = 14 $42 \div 3 = 14$
2-digit number divided by 1-digit number, with remainders	Use place value equipment to understand the concept of remainder. Make 29 from place value equipment. Share it into 2 equal groups. There are two groups of 14 and 1 remainder.	Use place value equipment to understand the concept of remainder in division. 29 ÷ 2 = ? 29 ÷ 2 = 14 remainder 1	Partition to divide, understanding the remainder in context. 67 children try to make 5 equal lines. 67 = 50 + 17 50 ÷ 5 = 10 17 ÷ 5 = 3 remainder 2 67 ÷ 5 = 13 remainder 2 There are 13 children in each line and 2 children left out.

		Year 4	
Year 4 Addition	Concrete	Pictorial	Abstract
Understanding numbers to 10,000	Use place value equipment to understand the place value of 4-digit numbers.	Represent numbers using place value counters once children understand the relationship between 1,000s and 100s. 2,000 + 500 + 40 + 2 = 2,542	Understand partitioning of 4-digit numbers, including numbers with digits of o. 5,000 + 60 + 8 = 5,068 Understand and read 4-digit numbers on a number line.
Choosing mental methods where appropriate	Use unitising and known facts to support mental calculations. <i>Make</i> 1,405 from place value equipment. Add 2,000. Now add the 1,000s. 1 thousand + 2 thousands = 3 thousands 1,405 + 2,000 = 3,405	Use unitising and known facts to support mental calculations. Th H T O O O O O O O O O O O O O O O O O O O	Use unitising and known facts to support mental calculations. 4,256 + 300 = ? 2 + 3 = 5 200 + 300 = 500 4,256 + 300 = 4,556

Column addition with exchange	Use place value equipment on a place value grid to organise thinking.	Use place value equipment to model required exchanges.	Use a column method to add, including exchanges.
	Ensure that children understand how the columns relate to place value and what to do if the numbers are not all 4-digit numbers. Use equipment.to show 1,905 + 775.		Th H T O I 5 5 4 + 4 2 3 7 I
	Th       H       T       O         Image: Constraint of the second row? Why is the Thousands box empty?       Image: Constraint of the second row?       Image: Constraint of the second row?	Th  H  T  O    Image: Second seco	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
	Which columns will total 10 or more?		$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
		Include examples that exchange in more than one column.	Th       H       T       O         I       5       5       4         +       4       2       3       7         5       7       9       I         I
			Include examples that exchange in more than one column.

Representing additions and checking strategies		Bar models may be used to represent additions in problem contexts, and to justify mental methods where appropriate. $\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	Use rounding and estimating on a number line to check the reasonableness of an addition.
Year 4 Subtraction	Concrete	Pictorial	Abstract
Choosing mental methods where appropriate	Use place value equipment to justify mental methods.	Use place value grids to support mental methods where appropriate. Th H T O Th H T O 7,646 - 40 = 7,606	Use knowledge of place value and unitising to subtract mentally where appropriate. 3,501 – 2,000 3 thousands – 2 thousands = 1 thousand 3,501 – 2,000 = 1,501



		$ \frac{\text{Th}}{2} \begin{array}{cccccccccccccccccccccccccccccccccccc$
Representing subtractions and checking strategies	Use bar models to represent subtractions where a part needs to be calculated. Total 5,762 ? ? Yes votes I can work out the total number of Yes votes using 5,762 – 2,899. Bar models can also represent 'find the difference' as a subtraction problem. Danny 899 Luis 1,005	Use inverse operations to check subtractions. I calculated 1,225 – 799 = 574. I will check by adding the parts. $ \begin{array}{r} \hline 1,225 \\ \hline 799 \\ \hline 799 \\ \hline 799 \\ \hline 799 \\ \hline 574 \\ \hline 1 \\ \hline 3 \\ \hline 7 \\ \hline 1 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline 7 \\ \hline 7 \\ \hline 9 \\ \hline 7 \\ \hline$

Year 4 Multiplication	Concrete	Pictorial	Abstract
Multiplying by multiples of 10 and 100	Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.	Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.	Use known facts and understanding of place value and commutativity to multiply mentally. 4 × 7 = 28
	3 groups of 4 ones is 12 ones. 3 groups of 4 tens is 12 tens. 3 groups of 4 hundreds is 12 hundreds.	3 × 4 = 12 3 × 40 = 120 3 × 400 = 1,200	4 × 70 = 280 40 × 7 = 280 4 × 700 = 2,800 400 × 7 = 2,800
Understanding times-tables up to 12 × 12	Understand the special cases of multiplying by 1 and 0.	Represent the relationship between the ×9 table and the ×10 table.	Understand how times-tables relate to counting patterns. Understand links between the ×3 table, ×6 table and ×9 table 5 × 6 is double 5 × 3
	5 × 1 = 5 5 × 0 = 0	Represent the ×11 table and ×12 tables in relation to the ×10 table.	<ul> <li>×5 table and ×6 table</li> <li><i>I</i> know that 7 × 5 = 35</li> <li>so I know that 7 × 6 = 35 + 7.</li> <li>×5 table and ×7 table</li> <li>3 × 7 = 3 × 5 + 3 × 2</li> </ul>
		$2 \times 11 = 20 + 2$ $3 \times 11 = 30 + 3$ $4 \times 11 = 40 + 4$	$3 \times 5 \qquad 3 \times 2$ $3 \times 5 \qquad 3 \times 2$ $3 \times 7$ $\times 9 \text{ table and } \times 10 \text{ table}$ $6 \times 10 = 60$
		4 × 12 = 40 + 8	6 × 9 = 60 - 6

Understanding and using partitioning in multiplication	Make multiplications by partitioning. 4 × 12 is 4 groups of 10 and 4 groups of 2.	Understand how multiplication and partitioning are related through addition. Understand how multiplication and partitioning are related through addition. Understand how multiplication and $0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\$	Use partitioning to multiply 2-digit numbers by a single digit. $18 \times 6 = ?$ $18 \times 6 = ?$ $18 \times 6 = 10 \times 6 + 8 \times 6$ $18 \times 6 = 10 \times 6 + 8 \times 6$ $18 \times 6 = 10 \times 6 + 8 \times 6$ $18 \times 6 = 10 \times 6 + 8 \times 6$ = 60 + 48 = 108
Column multiplication for 2- and 3-digit numbers multiplied by a single digit	Use place value equipment to make multiplications. Make 4 × 136 using equipment. Make 4 × 136 using equipment. Make 4 × 136 using equipment. Make 4 × 136 using equipment.	Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.	Use the formal column method for up to 3-digit numbers multiplied by a single digit. $3  1  2$ $\times  3$ $\boxed{9  3  6}$ Understand how the expanded column method is related to the formal column method and understand how any exchanges are related to place value at each stage of the calculation. $2  3$ $\frac{\times  5}{1  5} \qquad \frac{\times  5}{1  1  5}$ $\frac{1  0  0}{1  1  5}$

Multiplying more than two numbers	Represent situations by multiplying three numbers together.	Understand that commutativity can be used to multiply in different orders. 000000000000000000000000000000000000	Use knowledge of factors to simplify some multiplications. $24 \times 5 = 12 \times 2 \times 5$ $12 \times 2 \times 5 =$ $12 \times 10 = 120$ So, $24 \times 5 = 120$
Year 4 Division	Concrete	Pictorial	Abstract
Understanding the relationship between multiplication and division, including times- tables	Use objects to explore families of multiplication and division facts.	Represent divisions using an array.	Understand families of related multiplication and division facts. <i>I know that</i> 5 × 7 = 35 so <i>I know all these facts:</i> 5 × 7 = 35 7 × 5 = 35 35 = 5 × 7 35 = 7 × 5 35 ÷ 5 = 7 35 ÷ 7 = 5 7 = 35 ÷ 5 5 = 35 ÷ 7

Dividing multiples of 10 and 100 by a single digit	Use place value equipment to understand how to use unitising to divide. Use place value equipment to understand how to use unitising to divide.	Represent divisions using place value equipment. $q \div 3 = $ 1 1 1 1 1 1 1 1	Use known facts to divide 10s and 100s by a single digit. 15 ÷ 3 = 5 150 ÷ 3 = 50 1500 ÷ 3 = 500
Dividing 2-digit and 3-digit numbers by a single digit by partitioning into 100s, 10s and 1s	Partition into 10s and 1s to divide where appropriate. $39 \div 3 = ?$ $39 \div 3 = ?$ $39 \div 3 = ?$ $39 = 30 + 9$ $30 \div 3 = 10$ $9 \div 3 = 3$ $39 \div 3 = 13$	Partition into 100s, 10s and 1s using Base 10 equipment to divide where appropriate. $39 \div 3 = ?$ $39 \div 3 = ?$ $39 \Rightarrow 3 = ?$	Partition into 100s, 10s and 1s using a part- whole model to divide where appropriate. $142 \div 2 = ?$ $142 \div 2 = ?$ $1446 + 2 = 266 + 2 = $

Dividing 2-digit and 3-digit numbers by a	Use place value equipment to explore why different partitions are needed.	Represent how to partition flexibly where needed.	Make decisions about appropriate partitioning based on the division required.
single digit, using flexible	42 ÷ 3 = ?	84 ÷ 7 = ?	72 72 72 72
partitioning	I will split it into 30 and 12, so that I can divide by 3 more easily.	I will partition into 70 and 14 because I am dividing by 7.	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
		84 70 ÷ 7 = 10 84 ÷ 7 = 12	Understand that different partitions can be used to complete the same division. $ \begin{array}{c}  & & & \\ & & & & \\ & & & \\ & & & \\ & & & & \\ & & & \\ & $
Understanding remainders	Use place value equipment to find remainders.	Represent the remainder as the part that cannot be shared equally.	30+3=10 30+3=10 30+3=10 10+3=10 12+3=4 Understand how partitioning can reveal remainders of divisions.
	85 shared into 4 equal groups There are 24, and 1 that cannot be shared.		(95) (80) (15)
		72 ÷ 5 = 14 remainder 2	80 ÷ 4 = 20 12 ÷ 4 = 3 95 ÷ 4 = 23 remainder 3

## UPPER KEY STAGE 2

In upper Key Stage 2, children build on secure foundations in calculation, and develop fluency, accuracy and flexibility in their approach to the four operations. They work with whole numbers and adapt their skills to work with decimals, and they continue to develop their ability to select appropriate, accurate and efficient operations.

**Key language:** decimal, column methods, exchange, partition, mental method, ten thousand, hundred thousand, million, factor, multiple, prime number, square number, cube number

Addition and subtraction: Children build on their column methods to add and subtract numbers with up to seven digits, and they adapt the methods to calculate efficiently and effectively with decimals, ensuring understanding of place value at every stage. Children compare and contrast methods, and they select mental methods or jottings where appropriate and where these are more likely to be efficient or accurate when compared with formal column methods. Bar models are used to represent the calculations required to solve problems and may indicate where efficient methods can be chosen.	Multiplication and division: Building on their understanding, children develop methods to multiply up to 4-digit numbers by single-digit and 2- digit numbers. Children develop column methods with an understanding of place value, and they continue to use the key skill of unitising to multiply and divide by 10, 100 and 1,000. Written division methods are introduced and adapted for division by single-digit and 2-digit numbers and are understood alongside the area model and place value. In Year 6, children develop a secure understanding of how division is related to fractions. Multiplication and division of decimals are also introduced and refined in Year 6.	<b>Fractions:</b> Children find fractions of amounts, multiply a fraction by a whole number and by another fraction, divide a fraction by a whole number, and add and subtract fractions with different denominators. Children become more confident working with improper fractions and mixed numbers and can calculate with them. Understanding of decimals with up to 3 decimal places is built through place value and as fractions, and children calculate with decimals in the context of measure as well as in pure arithmetic. Children develop an understanding of percentages in relation to hundredths, and they understand how to work with common percentages: 50%, 25%, 10% and 1%.
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	Year 5			
Year 5 Addition	Concrete	Pictorial	Abstract	
Column addition with whole numbers	Use place value equipment to represent additions. Add a row of counters onto the place value grid to show 15,735 + 4,012.	Represent additions, using place value equipment on a place value grid alongside written methods. $\boxed{TTh Th H T O} OOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOO$	Use column addition, including exchanges. TTh Th H T O 1 9 1 7 5 + 1 8 4 1 7 3 7 5 9 2 	
Representing additions		Bar models represent addition of two or more numbers in the context of problem solving. $\begin{array}{c c} \hline & & & \\ \hline & & & $	Use approximation to check whether answers are reasonable. $\frac{TTh Th H T O}{2 3 4 0 5} + \frac{7 8 9 2}{2 0 2 9 7} + \frac{7 8 9 2}{3 1 2 9 7}$ I will use 23,000 + 8,000 to check.	

Adding tenths	Link measure with addition of decimals. Two lengths of fencing are 0.6 m and 0.2 m. How long are they when added together? 0.6 m 0.2 m	Use a bar model with a number line to add tenths. 0.6  m $0.2  m0.1  m 0.1  m 0.1  m 0.1  m 0.1  m 0.1  m0.1  m 0.1  m 0.1  m 0.1  m 0.1  m 0.1  m0.1  m 0.1  m 0.1  m 0.1  m 0.1  m 0.1  m0.1  m 0.1  m 0.1  m 0.1  m 0.1  m 0.1  m0.1  m 0.1  m 0.1  m 0.1  m 0.1  m 0.1  m0.1  m 0.1  m 0.1  m 0.1  m 0.1  m 0.1  m 0.1  m0.1  m 0.1  m$	Understand the link with adding fractions. $\frac{6}{10} + \frac{2}{10} = \frac{8}{10}$ 6 tenths + 2 tenths = 8 tenths 0.6 + 0.2 = 0.8
		6 tenths + 2 tenths = 8 tenths	
Adding decimals using column addition	Use place value equipment to represent additions.	Use place value equipment on a place value grid to represent additions.	Add using a column method, ensuring that children understand the link with place value.
	Show 0.23 + 0.45 using place value counters.	Represent exchange where necessary. $\bigcirc$ $\boxed{Tth}$ $\boxed{Hth}$ $\bigcirc$ $\bigcirc$ $\boxed{0 \cdot Tth}$ $\underbrace{Hth}$ $\bigcirc$ <td< th=""><th><math display="block">\frac{O \cdot \text{Tth Hth}}{0 \cdot 2 \cdot 3}</math> + <math>\frac{O \cdot 4 \cdot 5}{0 \cdot 6 \cdot 8}</math> Include exchange where required, alongside an understanding of place value. <math display="block">\frac{O \cdot \text{Tth Hth}}{0 \cdot 9 \cdot 2}</math> + <math>\frac{O \cdot 3 \cdot 3}{1 \cdot 2 \cdot 5}</math> Include additions where the numbers of decimal places are different. 3.4 + 0.65 = ? <math display="block">\frac{O \cdot \text{Tth Hth}}{3 \cdot 4 \cdot 0}</math> + <math>\frac{O \cdot 6 \cdot 5}{2}</math></th></td<>	$\frac{O \cdot \text{Tth Hth}}{0 \cdot 2 \cdot 3}$ + $\frac{O \cdot 4 \cdot 5}{0 \cdot 6 \cdot 8}$ Include exchange where required, alongside an understanding of place value. $\frac{O \cdot \text{Tth Hth}}{0 \cdot 9 \cdot 2}$ + $\frac{O \cdot 3 \cdot 3}{1 \cdot 2 \cdot 5}$ Include additions where the numbers of decimal places are different. 3.4 + 0.65 = ? $\frac{O \cdot \text{Tth Hth}}{3 \cdot 4 \cdot 0}$ + $\frac{O \cdot 6 \cdot 5}{2}$

Year 5 Subtraction	Concrete	Pictorial	Abstract
Column subtraction with whole numbers	Use place value equipment to understand where exchanges are required. 2,250 – 1,070	Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required. 15,735 - 2,582 = 13,153 $\begin{array}{rrrr} \hline Th & H & T & 0 \\ \hline 1 & 5 & 7 & 3 & 5 \\ \hline 2 & 5 & 8 & 2 \\ \hline 3 \\ \hline \end{array}$ Now subtract the l0s. Exchange I hundred for I0 tens. $\begin{array}{rrrr} \hline Th & H & T & 0 \\ \hline 1 & 5 & 7' & 3 & 5 \\ \hline 2 & 5 & 8 & 2 \\ \hline 3 \\ \hline \end{array}$ Now subtract the l0s. I,000s and I0,000s. $\begin{array}{rrrr} \hline Th & Th & H & T & 0 \\ \hline 1 & 5 & 5' & 3 & 5 \\ \hline 2 & 5 & 8 & 2 \\ \hline 5 & 3 \\ \hline \end{array}$	Use column subtraction methods with exchange where required. $\frac{TTh Th H T O}{\frac{5}{8} \frac{11}{2} 0 q 7}$ $-\frac{1 8 5 3 4}{4 3 5 6 3}$ $62,097 - 18,534 = 43,563$
Checking strategies and representing subtractions		Bar models represent subtractions in problem contexts, including 'find the difference'. Athletics Stadium 75,450 Hockey Centre 42,300 Velodrome 15,735 ?	Children can explain the mistake made when the columns have not been ordered correctly. $ \begin{array}{r} \hline Th Th H T 0 \\ \hline \hline 1 7 8 7 7 \\ + \frac{4 0 1 2}{5 7 9 9 7} \\ \hline \end{array} $ Use approximation to check calculations. I calculated 18,000 + 4,000 mentally to check my subtraction.

Choosing efficient methods			To subtract two large numbers that are close, children find the difference by counting on. 2,002 - 1,995 = ? 45 + 5 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 +
Subtracting decimals	Explore complements to a whole number by working in the context of length. 0.49 m 1 m - m = m 1 - 0.49 = ?	Use a place value grid to represent the stages of column subtraction, including exchanges where required. $5\cdot74 - 2\cdot25 = ?$ $\bigcirc & Th & Hth \\ \hline \odot & \odot$	Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places. 3·921 - 3·75 = ? O · Tth Hth Thth 3 · 9 2 1 - 3 · 7 5 0

Year 5 Multiplication	Concrete	Pictorial	Abstract
Understanding factors	Use cubes or counters to explore the meaning of 'square numbers'.	Use images to explore examples and non- examples of square numbers.	Understand the pattern of square numbers in the multiplication tables.
	25 is a square number because it is made from 5 rows of 5. Use cubes to explore cube numbers.		Use a multiplication grid to circle each square number. Can children spot a pattern?
		$8 \times 8 = 64$ $8^2 = 64$	
	8 is a cube number.	12 is not a square number, because you cannot multiply a whole number by itself to make 12.	
Multiplying by 10, 100 and 1,000	Use place value equipment to multiply by 10, 100 and 1,000 by unitising.	Understand the effect of repeated multiplication by 10.	Understand how exchange relates to the digits when multiplying by 10, 100 and 1,000.
	$4 \times 1 = 4 \text{ ones} = 4$ 555 $4 \times 10 = 4 \text{ tens} = 40$ $4 \times 100 = 4 \text{ hundreds}$		H T O I 7 17 × 10 = 170 17 × 100 = 17 × 10 × 10 = 1,700 17 × 1,000 = 17 × 10 × 10 × 10 = 17,000

Multiplying by multiples of 10, 100 and 1,000	Use place value equipment to explore multiplying by unitising.	Use place value equipment to represent how to multiply by multiples of 10, 100 and 1,000.	Use known facts and unitising to multiply. 5 × 4 = 20 5 × 40 = 200 5 × 400 = 2,000 5 × 4,000 - 20,000
	5 groups of 3 ones is 15 ones. 5 groups of 3 tens is 15 tens. So, I know that 5 groups of 3 thousands would be 15 thousands.	4 × 3 = 12       6 × 4 = 24         4 × 300 = 1,200       6 × 400 = 2,400	5,000 × 4 = 20,000
Multiplying up to 4-digit numbers by a single digit	Explore how to use partitioning to multiply efficiently. $8 \times 17 = ?$ $8 \times 10 = 80$ $8 \times 10 = 80$ $8 \times 7 = 56$ 80 + 56 = 136	Represent multiplications using place value equipment and add the 1s, then 1os, then 10os, then 1,000s.	Use an area model and then add the parts. $100  60  3$ $5  100 \times 5 = 500  60 \times 5 = 300  3 \times 5 = 15$ Use a column multiplication, including any required exchanges. $1  3  6$ $\times \qquad 6$ $\boxed{8  1  6}{2  3}$

Multiplying 2- digit numbers by 2-digit numbers	Partition one number into 10s and 1s, then add the parts. $23 \times 15 = ?$ $10 \times 15 = 150$ $10 \times 15 = 150$ $1 \times 15 = 1$	Use an area model and add the parts. $28 \times 15 = ?$ 10 m $20 \times 10 = 200 \text{ m}^2$ 5  m $20 \times 5 = 100 \text{ m}^2$ $8 \times 5 = 40 \text{ m}^2$ $8 \times 5 = 40 \text{ m}^2$ $8 \times 15 = 420$ $\frac{H \text{ T O}}{2 \text{ 0 0}}$ 1  0 0 8  0 $+ \frac{4 \text{ 0}}{4 \text{ 2 0}}$	Use column multiplication, ensuring understanding of place value at each stage. $3 4$ $\times 27$ $23_{2}8 34 \times 7$ $3 4$ $\times 27$ $23_{2}8 34 \times 7$ $6 8 0 34 \times 20$ $3 4$ $\times 27$ $23_{2}8 34 \times 7$ $6 8 0 34 \times 20$ $3 4$
Multiplying up to 4-digits by 2- digits		Use the area model then add the parts. $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Use column multiplication, ensuring understanding of place value at each stage. $ \begin{array}{r}                                     $

			$ \begin{array}{c}                                     $
Multiplying decimals by 10, 100 and 1,000	Use place value equipment to explore and understand the exchange of 10 tenths, 10 hundredths or 10 thousandths.	Represent multiplication by 10 as exchange on a place value grid.	Understand how this exchange is represented on a place value chart.         Th       H       T       O       Tth         2·5 × 10 = 25       2       5       0       2         2·5 × 100 = 250       2       5       0       0       0         2·5 × 1,000 = 2,500       2       5       0       0       0       0

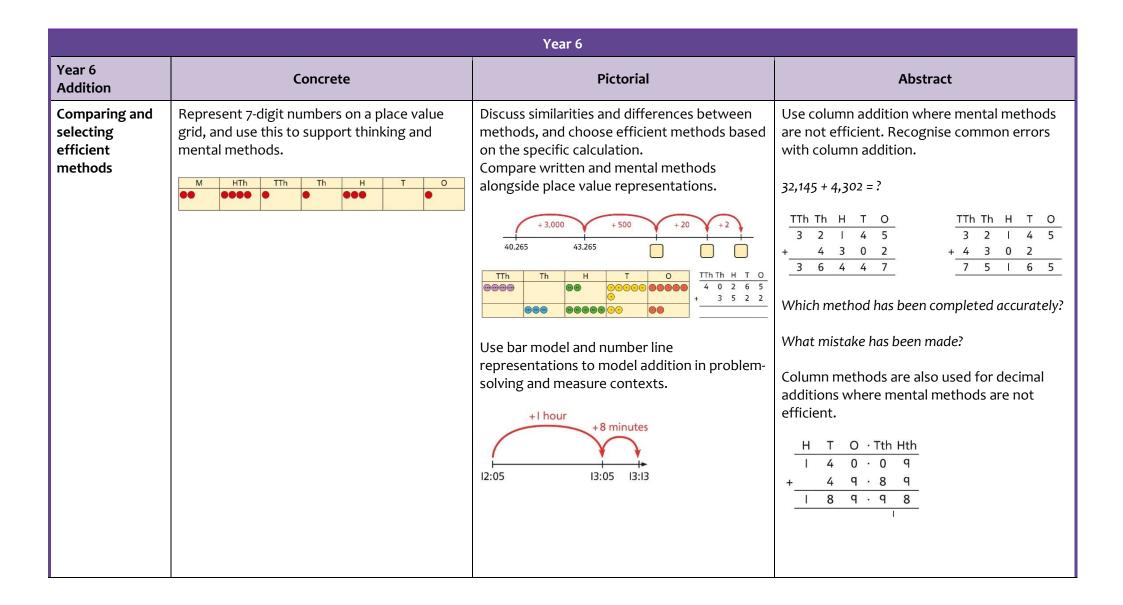
Year 5 Division	Concrete	Pictorial	Abstract
Understanding factors and prime numbers	Use equipment to explore the factors of a given number.	Understand that prime numbers are numbers with exactly two factors.	Understand how to recognise prime and composite numbers.
	24 ÷ 3 = 8 24 ÷ 8 = 3 8 and 3 are factors of 24 because they divide 24 exactly.	13 ÷ 1 = 13 13 ÷ 2 = 6 r 1 13 ÷ 4 = 4 r 1 1 and 13 are the only factors of 13. 13 is a prime number.	I know that 31 is a prime number because it can be divided by only 1 and itself without leaving a remainder. I know that 33 is not a prime number as it can be divided by 1, 3, 11 and 33.
	24 ÷ 5 = 4 remainder 4. 5 is not a factor of 24 because there is a remainder.		I know that 1 is not a prime number, as it has only 1 factor.
Understanding inverse operations and	Use equipment to group and share and to explore the calculations that are present.	Represent multiplicative relationships and explore the families of division facts.	Represent the different multiplicative relationships to solve problems requiring inverse operations.
the link with multiplication, grouping and sharing	I have 28 counters. I made 7 groups of 4. There are 28 in total. I have 28 in total. I shared them equally into 7 groups. There are 4 in each group. I have 28 in total. I made groups of 4. There are 7 equal groups.	60 ÷ 4 = 15 60 ÷ 15 = 4	$12 \div 3 = 12$ $12 \div 3 = 12$ $12 \div 3 = 12$ $3 = 12$ $3 = 12$ Understand missing number problems for division calculations and know how to solve them using inverse operations. $22 \div 2 = 2$

Dividing whole numbers by 10, 100 and 1,000	Use place value equipment to support unitising for division. 4,000 ÷ 1,000	Use a bar model to support dividing by unitising. 380 ÷ 10 = 38	Understand how and why the digits change on a place value grid when dividing by 10, 100 or 1,000.
	$4,000 \times 1,000 \times 1,000 = 4$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Th       H       T       O         3       2       0       0 $3,200 \div 100 = ?$ $3,200 \div 100 = 2$ $3,000 \div 100 = 30$ $3,200 \div 100 = 32$ $3,200 \div 100 = 32$ So, the digits will move two places to the right.
Dividing by multiples of 10, 100 and 1,000	Use place value equipment to represent known facts and unitising.	So, $380 \div 10 = 38$ Represent related facts with place value equipment when dividing by unitising. 180 is 18 tens. 18 tens divided into groups of 3 tens. There are 6 groups. 180 $\div 30 = 6$ 100 100 100 100 100 100 100 100 100 100 100 100 100 100	Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check. $3,000 \div 5 = 600$ $3,000 \div 50 = 60$ $5 \times 600 = 3,000$ $50 \times 60 = 3,000$ $500 \times 6 = 3,000$

		12 ones divided into groups of 4. There are 3 groups. 12 hundreds divided into groups of 4 hundreds. There are 3 groups. 1200 ÷ 400 = 3	
Dividing up to four digits by a single digit using short division	Explore grouping using place value equipment. 268 ÷ 2 = ? There is 1 group of 2 hundreds. There are 3 groups of 2 tens. There are 4 groups of 2 ones. 264 ÷ 2 = 134	Use place value equipment on a place value grid alongside short division. The model uses grouping. A sharing model can also be used, although the model would need adapting. $4 \boxed{4 \ 8} \qquad \boxed{T \ 0} \\ 4 \boxed{4 \ 8} \qquad \boxed{T \ 0} \\ 6 \boxed{6 \ 0} \\ 6 \boxed$	Use short division for up to 4-digit numbers divided by a single digit. 0 5 5 6 7 3 38 3 9 42 $3,892 \div 7 = 556$ Use multiplication to check. $556 \times 7 = ?$ $6 \times 7 = 42$ $50 \times 7 = 350$ $500 \times 7 = 3500$ 3,500 + 350 + 42 = 3,892

		4 $\overline{q}$ $\overline{1}$ $\overline{0}$ First, lay out the problem.4 $\overline{q}$ $\overline{2}$ $\overline{1}$ $\overline{0}$ $\overline{0}$ 4 $\overline{q}$ $\overline{2}$ $\overline{1}$ $\overline{0}$ $\overline{0}$ 3groups of 4 ones. $\overline{3}$ $\overline{3}$
Understanding remainders	Understand remainders using concrete versions of a problem. 80 cakes divided into trays of 6. 80 cakes in total. They make 13 groups of 6, with 2 remaining.	Use short division and understand remainders as the last remaining 1s. $ \begin{array}{c c} \hline & \hline \\ \\ & \hline \\ \\ & \hline \\ & \hline \\ \\ \\ & \hline \\ \\ & \hline \\ \\ \\ \\$

Dividing decimals by 10, 100 and 1,000	Understand division by 10 using exchange.	Represent division using exchange on a place value grid.	Understand the movement of digits on a place value grid.
	2 ones are 20 tenths. 20 tenths divided by 10 is 2 tenths.	0       Tth       Hth         0       This is called by 10 is 1 tenths.       This is equivalent to 10 tenths and 50 hundredths.         10 tenths divided by 10 is 1 tenth.       50 hundredths divided by 10 is 5 hundredths.         10 tenths divided by 10 is 1 tenth and 5 hundredths.       1.5 divided by 10 is 1 tenth and 5 hundredths.         1.5 till       To 10 tenth       Tenth	$0 \cdot 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$
Understanding the relationship between fractions and division	Use sharing to explore the link between fractions and division. 1 whole shared between 3 people. Each person receives one-third.	Use a bar model and other fraction representations to show the link between fractions and division. $I \div 3 = \frac{1}{3}$	Use the link between division and fractions to calculate divisions. $5 \div 4 = \frac{5}{4} = 1\frac{1}{4}$ $11 \div 4 = \frac{11}{4} = 2\frac{3}{4}$



Selecting mental methods for larger numbers where	Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods.	Use a bar model to support thinking in addition problems. 257,000 + 99,000 = ?	Use place value and unitising to support mental calculations with larger numbers. 195,000 + 6,000 = ?
appropriate	M       HTh       Th       Th       H       T       O         2,411,301 + 500,000 = ?         This would be 5 more counters in the HTh place.         So, the total is 2,911,301.         2,411,301 + 500,000 = 2,911,301	$\begin{array}{c c} 257,000 + 99,000 = 1 \\ \hline \\$	195,000 + 0,000 - : 195 + 5 + 1 = 201 195 thousands + 6 thousands = 201 thousands So, 195,000 + 6,000 = 201,000
Understanding order of operations in calculations	Use equipment to model different interpretations of a calculation with more than one operation. Explore different results. $3 \times 5 - 2 = ?$	Model calculations using a bar model to demonstrate the correct order of operations in multi-step calculations. $16 \times 4$ cab $444444444444444444444444444444444444$	Understand the correct order of operations in calculations without brackets. Understand how brackets affect the order of operations in a calculation. $4 + 6 \times 16$ 4 + 96 = 100 $(4 + 6) \times 16$ $10 \times 16 = 160$

Year 6 Subtraction	Concrete	Pictorial	Abstract
Comparing and selecting efficient methods	Use counters on a place value grid to represent subtractions of larger numbers.	Compare subtraction methods alongside place value representations. $\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	Compare and select methods. Use column subtraction when mental methods are not efficient. Use two different methods for one calculation as a checking strategy. $\frac{\frac{Th}{1} + \frac{H}{97} + \frac{T}{92}}{\frac{1}{97} + \frac{5}{9} + \frac{6}{1,552} + \frac{-400}{1,552} + \frac{6}{1,552} + \frac{1}{1,952}}$ Use column subtraction for decimal problems, including in the context of measure. $\frac{H}{1} + \frac{T}{9} + \frac{O}{1} + \frac{O}{1} + \frac{1}{1,952} + \frac{O}{1,952} + \frac{1}{1,952} + $
Subtracting mentally with larger numbers		Use a bar model to show how unitising can support mental calculations. 950,000 – 150,000 That is 950 thousands – 150 thousands $950 \\ 150 \\ 800 \\ 800 \\ 50, the difference is 800 thousands.950,000 – 150,000 = 800,000$	Subtract efficiently from powers of 10. 10,000 - 500 = ?

Year 6 Multiplication	Concrete	Pictorial	Abstract
Multiplying up to a 4-digit number by a single digit number	Use equipment to explore multiplications.	Use place value equipment to compare methods. Method I	Understand grid method and short multiplication. Compare and select appropriate methods for specific multiplications.
	4 groups of 2,345 This is a multiplication: 4 × 2,345 2,345 × 4	$3 2 2 5$ $+ 3 2 2 5$ $1 2 9 0 0$ $1 1 2$ $Method 2$ $4 \times 3.000 4 \times 200 4 \times 20 4 \times 5$ $12.000 + 800 + 80 + 20 = 12.900$	Method 3 $3,000 \ 200 \ 20 \ 5$ $4 \ 12,000 \ 800 \ 80 \ 20$ 12.000 + 800 + 80 + 20 = 12,900 Method 4 $3 \ 2 \ 2 \ 5$ $\times \ 4$ $1 \ 2 \ 9 \ 0 \ 0$
Multiplying up to a 4-digit number by a 2-digit number		Use an area model alongside written multiplication. Method I 1,000  200  30  5 20  20,000  4,000  600  100 1  1,000  200  30  5 $\times  2  1$ $5  1 \times 5$ $3  0  1 \times 30$ $2  0  0  1 \times 1000$ $1  0  0  20 \times 5$ $6  0  0  20 \times 30$ $4  0  0  20 \times 200$ $2  0  0  0  20 \times 1,000$ 2  5  9  3  5 2  9  3  5	Use long multiplication with understanding of place value at all stages. $ \begin{array}{r} 1 & 2 & 3 & 5 \\ \times & 2 & 1 \\ \hline 1 & 2 & 3 & 5 \\ \hline 2 & 4 & 7 & 0 & 0 \\ \hline 2 & 5 & 9 & 3 & 5 \\ \hline \end{array} $ $ \begin{array}{r} 1 \times 1,235 \\ \hline 2 & 5 & 9 & 3 & 5 \\ \hline 2 & 5 & 9 & 3 & 5 \\ \hline \end{array} $

Using knowledge of factors and partitions to compare methods for multiplications	Use equipment to understand square numbers and cube numbers. $5 \times 5 = 5^2 = 25$ $5 \times 5 = 5^3 = 25 \times 5 = 125$	Compare methods visually using an area model. Understand that multiple approaches will produce the same answer if completed accurately.	Use a known fact to generate families of related facts. $170 \times 11$ $171 \times 11$ $171 \times 11$ $171 \times 11$ $170 \times 12$ $17 \times 110$ Use factors to calculate efficiently. $15 \times 16$ $= 3 \times 5 \times 2 \times 8$ $= 3 \times 8 \times 2 \times 5$ $= 24 \times 10$ $= 240$
Multiplying by 10, 100 and 1,000	Use place value equipment to explore exchange in decimal multiplication. $ \begin{array}{c c} \hline \hline 0 & \hline \\ \hline \hline \hline 0 & \hline \\ \hline \hline \hline 0 & \hline \\ \hline \hline$	Understand how the exchange affects decimal numbers on a place value grid. $\boxed{T \ 0 \ T \ 0} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ $	Use knowledge of multiplying by 10, 100 and 1,000 to multiply by multiples of 10, 100 and 1,000. $8 \times 100 = 800$ $8 \times 300 = 800 \times 3$ = 2,400 $2.5 \times 10 = 25$ $2.5 \times 20 = 2.5 \times 10 \times 2$ = 50

Multiplying decimals	Explore decimal multiplications using place value equipment and in the context of	Represent calculations on a place value grid.	Use knov	wn fac	ts to	multi	ply o	decim	als.
	measures.	3 × 3 = 9	4 × 3 = 12						
		$3 \times 0.3 = 0.9$	4 × 0·3 = 1	1•2					
		$3 \times 0.3 = 0.4$	4 × 0·03 =	= 0 <b>·</b> 12					
		T O • Tth							
			20 × 5 = 10						
			20 × 0·5 =						
			20 × 0∙05	= 1					
	3 groups of 4 tenths is 12 tenths.		Find fami	ilies o	f fact	s fron	n a k	nowr	า
	4 groups of 3 tenths is 12 tenths.	Understand the link between multiplying	multiplica	ation.					
		decimals and repeated addition.							
			I know th	nat 18 :	× 4 = 7	72.			
	·3 cm l·3 cm l·3 cm l·3 cm		This can h	help m	ne wo	rk out	:		
	4 × 1 cm = 4 cm	0 I	1.8 × 4 = ?	,					
	$4 \times 0.3$ cm = 1.2 cm		18 × 0·4 =						
	4 × 1·3 = 4 + 1·2 = 5·2 cm		180 × 0·4						
			18 × 0·04						
			Use a pla	ace val	lue gr	id to	unde	erstar	nd the
			effects of						
			Г	Н	Т	0	•	Tth	Hth
			2 × 3			6			
			-						
			0·2 × 3			0	•	6	
			0·02 × 3				•		

Year 6 Division	Concrete	Pictorial	Abstract
Understanding factors	Use equipment to explore different factors of a number.	Recognise prime numbers as numbers having exactly two factors. Understand the link with division and remainders.	Recognise and know primes up to 100. Understand that 2 is the only even prime, and that 1 is not a prime number.
	$24 \div 4 = 6$ $30 \div 4 = 7 \text{ remainder } 2$ 4 is a factor of 24 but is not a factor of 30.	I7 ÷ 2 = 8 r I       I7 ÷ 3 = 5 r 2       I7 ÷ 4 = 4 r I       I7 ÷ 5 = 3 r 2	I       2       3       4       5       6       7       8       9       10         II       I2       I3       I4       I5       I6       17       18       19       20         21       22       23       24       25       26       27       28       29       30
Dividing by a single digit	Use equipment to make groups from a total.	H T O Grave and the equation of the equation	Use short division to divide by a single digit. $ \begin{array}{c} 0\\ 6\\ 1 \\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 6\\ 1 \\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 6\\ 1 \\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 6\\ 1 \\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 6\\ 1 \\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 6\\ 1 \\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 2\\ 1 \\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 2\\ 2\\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 2\\ 2\\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 2\\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 2\\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 2\\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 2\\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 2\\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 2\\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 2\\ 2\\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 2\\ 2\\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 2\\ 2\\ 2\\ 3 \\ 2 \end{array} $ $ \begin{array}{c} 0\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\ 2\\$

Dividing by a 2- digit number using factors	Understand that division by factors can be used when dividing by a number that is not prime.	Use factors and repeated division. $1,260 \div 14 = ?$ 1,260 $1,260 \div 2 = 630$ $630 \div 7 = 90$ $1,260 \div 14 = 90$	Use factors and repeated division where appropriate. 2,100 ÷ 12 = ? 2,100 $\rightarrow$ (*2) $\rightarrow$ (*6) $\rightarrow$ 2,100 $\rightarrow$ (*6) $\rightarrow$ (*2) $\rightarrow$ 2,100 $\rightarrow$ (*6) $\rightarrow$ (*2) $\rightarrow$ 2,100 $\rightarrow$ (*3) $\rightarrow$ (*4) $\rightarrow$ 2,100 $\rightarrow$ (*3) $\rightarrow$ (*4) $\rightarrow$ 2,100 $\rightarrow$ (*3) $\rightarrow$ (*2) $\rightarrow$ (*2) $\rightarrow$
Dividing by a 2- digit number using long division	Use equipment to build numbers from groups. 182 divided into groups of 13. There are 14 groups.	Use an area model alongside written division to model the process. $377 \div 13 = ?$ $13 \ 377 \ 13 \ 247 \ 30 \ 247 \ 30 \ 130 \ 177 \ 30 \ 300 \ 177 \ 300 \ 300 \ 177 \ 377 \div 13 = 29$	Use long division where factors are not useful (for example, when dividing by a 2-digit prime number). Write the required multiples to support the division process. $377 \div 13 = ?$ $\overrightarrow{0}  \cancel{13}  \cancel{26}  \cancel{39}  \cancel{52}  \cancel{65}  \cancel{78}  \cancel{91}  \cancel{104}  \cancel{117}  \cancel{130}  \cancel{9\times13}  \cancel{10\times13}  \cancel{9\times13}  \cancel{9\times13}  \cancel{10\times13}  \cancel{9\times13}  \cancel{10\times13}  \cancel{9\times13}  \cancel{10\times13}  \cancel{9\times13}  \cancel{10\times13}  \cancel{9\times13}  \cancel{10\times13}  \cancel{9\times13}  \cancel{9\times13}  \cancel{10\times13}  \cancel{9\times13}  \cancel{9\times13}$

			$3$ $21 \overline{7 \ 9 \ 8}$ $- \frac{6 \ 3 \ 0}{1 \ 6 \ 8}$ $21 \overline{7 \ 9 \ 8}$ $- \frac{3 \ 8}{1 \ 6 \ 8}$ $- \frac{6 \ 3 \ 0}{1 \ 6 \ 8}$ $- \frac{6 \ 3 \ 0}{1 \ 6 \ 8}$ $- \frac{1 \ 6 \ 8}{0}$ Divisions with a remainder explored in problem-solving contexts.
Dividing by 10, 100 and 1,000	Use place value equipment to explore division as exchange. Image: Image: Image and the problem of the problem	Represent division to show the relationship with multiplication. Understand the effect of dividing by 10, 100 and 1,000 on the digits on a place value grid. $\begin{array}{c c c c c c c c c c c c c c c c c c c $	Use knowledge of factors to divide by multiples of 10, 100 and 1,000. $40 \div 50 = 10$ $40 \rightarrow \div 10 \rightarrow \div 5 \rightarrow ?$ $40 \rightarrow \div 5 \rightarrow \div 10 \rightarrow ?$ $40 \div 5 = 8$ $8 \div 10 = 0.8$ $50, 40 \div 50 = 0.8$

Dividing decimals	Use place value equipment to explore division of decimals.	Use a bar model to represent divisions.	Use short division to divide decimals with up to 2 decimal places.
	8 tenths divided into 4 groups. 2 tenths in each group.	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	$8 \overline{4 \cdot 2 4}$ $8 \overline{4 \cdot 42 4}$ $8 \overline{4 \cdot 42 4}$ $0 \cdot 5$ $8 \overline{4 \cdot 42 24}$ $0 \cdot 5$ $8 \overline{4 \cdot 42 24}$ $0 \cdot 5 3$ $8 \overline{4 \cdot 42 24}$